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FORM-FINDING OF SHELL AND MEMBRANE STRUCTURES

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Summary. This paper deals with the form-finding process of building constructions, such as membrane, shell or cable structures. This task is an essential step in the design process of membrane or cable structures, while for shell or beam structures, it is an optional case how to prefer the axial behavior of the intended construction. As a result of the form-finding process for structures acting exclusively in tension or compression, one unique shape in equilibrium is calculated for such a requirement. In the case of combined structures, cases with more than one solution for the same form-finding definition may occur. If the form-finding process is defined as a continuum mechanics process of the energy extreme calculation, the resulting shapes can reach both stable and unstable equilibrium positions. This contribution will focus on the general behavior of the process mentioned above, particular statements will be accompanied by examples.

1 INTRODUCTION

Lightweight constructions are characterized by their optimal material utilization since the axial stresses are preferred to bending action. For tensile membrane structures, this is a natural way of acting because the textile or foil can virtually resist in tension only. Such a strong restriction is not required for shell structures; however, this construction optimization by minimizing bending action would provide the possibility to design slender and beautiful shapes [1]. The optimization process may also face the task of combined structures, where compressed arches are in the interaction with tensioned membranes as an example.

Shapes of the lightweight structures are interconnected with their acting and there is a need of their calculation. This problem is well-known as form-finding, where the shape is a result and internal axial forces are an input. This analysis shows some phenomena as the consequences of such an inverse process [2-6]. In the form-finding analysis, the external load and boundary conditions are the shaping parameters of the structure as well as internal forces. In order to solve this problem, certain methods have been developed to overcome or even avoid the natural singularity of such a process [2-13].

Regardless the particular method, the aim of the form-finding process is to calculate the shape in equilibrium. The resulting shape can physically take both stable or unstable equilibrium position as both cases satisfy the extreme of the energy.

2 FORM-FINDING IN CONTEXT OF CONTINUUM MECHANICS

There are different ways of how to formulate the form-finding analysis, as this can be defined as a mathematical process or as a physical process. The most general methods are based on the continuum mechanics conception. The solution is derived from the energetic concept, generally from the variational formulation of the problem, where it is search for an extreme of the operator Π that is of an additive nature. Here, the potential energy $\Pi = \Pi_{int} + \Pi_{ext}$ of the internal and external forces in the body is - according to the Lagrange variational principle - minimal just for the real state of the body (d, ε, σ) [14]. The general FEM equations can be obtained from the differentiation of Π with respect to the deformation d (1).

$$\frac{\partial \Pi}{\partial d} = \frac{\partial(\Pi_{int} + \Pi_{ext})}{\partial d} = \frac{\partial \Pi_{int}}{\partial d} + \frac{\partial \Pi_{ext}}{\partial d} = \boldsymbol{K}d - f = 0$$

The equilibrium shape is identified when the $\partial\Pi/\partial d = 0$, thus the integral of the internal and external energy increment over the domain Ω has to be zero (2). Where $\boldsymbol{\sigma}$ is the Cauchy stress tensor acting in the structure, $\delta\hat{\boldsymbol{e}}$ is a variation of the Euler-Almansi strain tensor, $\vec{p}$ is a load, and δd is an increment of deformation.

$$\frac{\partial \Pi}{\partial d} = \frac{\partial \Pi_{int}}{\partial d} + \frac{\partial \Pi_{ext}}{\partial d} = \int_{\Omega} \boldsymbol{\sigma} : \delta\hat{\boldsymbol{e}}\, d\Omega - \int_{\Omega} \vec{p} \cdot \delta d\, d\Omega = 0$$

Even though the spatial shape is given by the equilibrium prestress, eventually considering the load, the direct numerical solution of such a problem is not possible. This fact is described in [6] and followed by the illustration of such a singularity (Figure 1). Such a singularity has to be overcome, therefore different methods of the form-finding were proposed for such a problem [13].

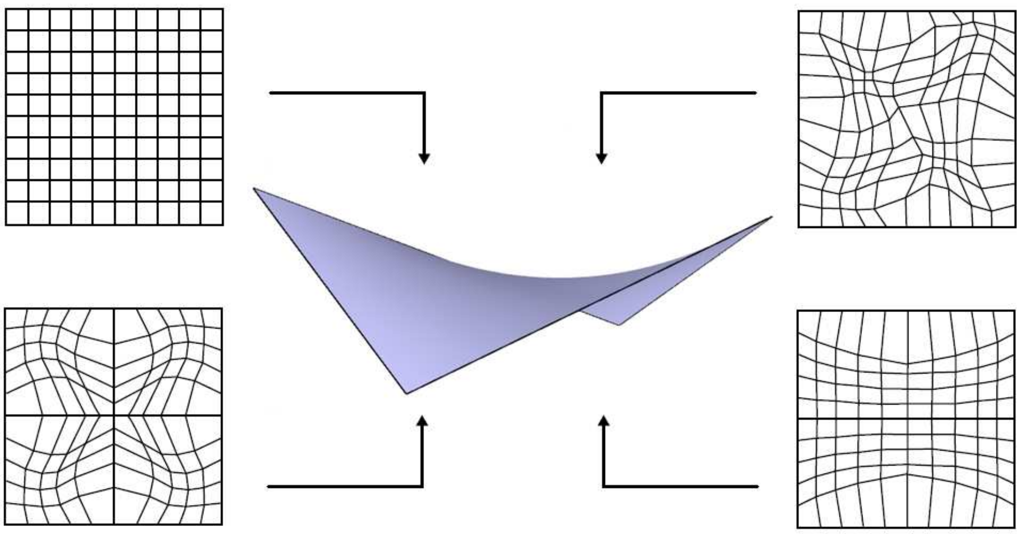

Figure 1: Arbitrarily deformed meshes for the same surface geometry ([6] with modification)

3 FORCE-FINDING AS A PART OF FORM-FINDING

Generally, there is one more phenomena to face in the form-finding process, which is the finding of the equilibrium prestress itself. The only homogenous prestress which can exist for surfaces with nonzero Gaussian curvature is an isotropic prestress. The homogenous orthotropic prestress is not in equilibrium for double-curved surfaces so the second task is to find the equilibrium prestress itself, which is derived from the prescribed values in warp and weft in some way. There are many methods proposed for such a stabilization; some of them are based on changing the force according to the deformation while other are derived from the restriction of the maximum allowable deformation of finite elements [5,9]. Generally, it is desirable to find as close approximation of the prescribed values as possible without objectionable concentrations.

4 SEARCHING FOR UNSTABLE EQUILIBRIUM POSITION OF ELEMENTS IN COMPRESSION

There is one important difference in the form-finding process of the tensioned and compressed structures or structural parts. While the tensioned elements take the stable equilibrium position in the form-finding process, the compressed elements are forced to take the unstable equilibrium position after the calculation. Such a phenomenon has to be overcome to reach the resulting equilibrium shape after the form-finding process.

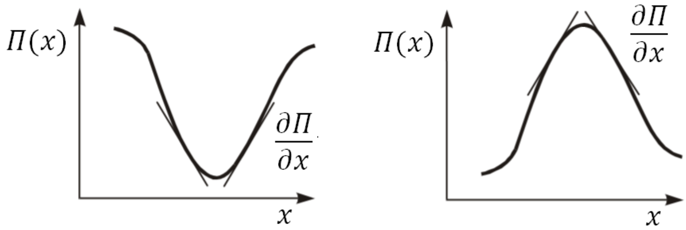

Figure 2: Stable and unstable equilibrium position

This fact can be illustrated on the example of a steel cable and a concrete arch (Figure 3). If only axial stresses are allowed in those elements, any deviation from the equilibrium position of an arbitrary node on the arch shifts the structure away from this equilibrium position.

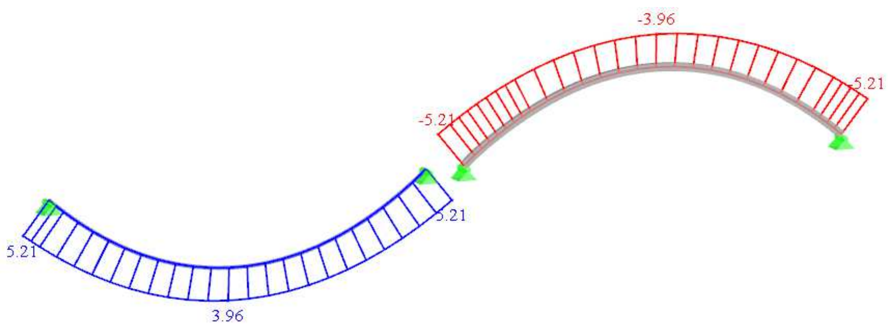

Figure 3: Equilibrium shapes of cable and arch

The well-known way of how to overcome this behavior is to replace the compressed elements by tensioned elements with inverted loads. Such a method, already used in ancient time for physical models, provides strong capabilities for the form-finding of exclusively compressed structures. However, this method is not applicable for structures combining both tensioned and compressed elements, since the inversion of the compressed elements into the tensioned ones would lead to the conversion of the tensioned elements into the compressed ones. The local stabilization has to be implemented for finding the unstable equilibrium positions of the compressed elements while the stable equilibrium positions of the tensioned elements are calculated (Figure 7-8).

5 FORM-FINDING OF COMBINED STRUCTURES AND ITS PHENOMENA

This chapter presents the form-finding process of combined structures as well as one more phenomena of this analysis. The following pictures (Figures 4-8) show a membrane structure with boundary cables, supported by arches. First, the form-finding was only performed for the tensioned parts of the structure (Figures 5-6) and then for the whole structure including the arches (Figures 7-8).

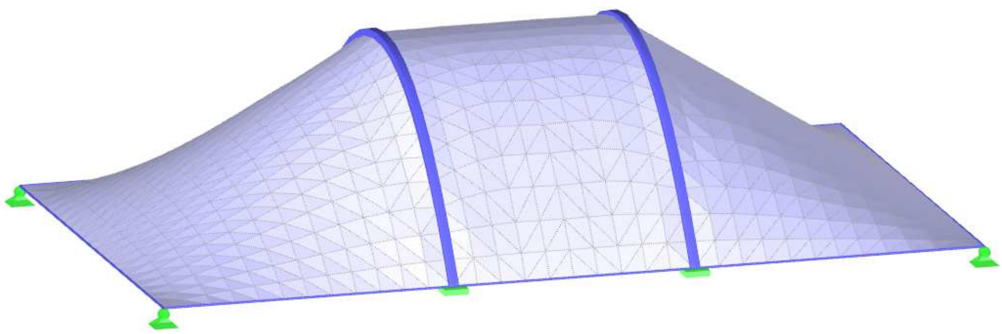

Figure 4: Initial position and FE mesh of the membrane supported by arches

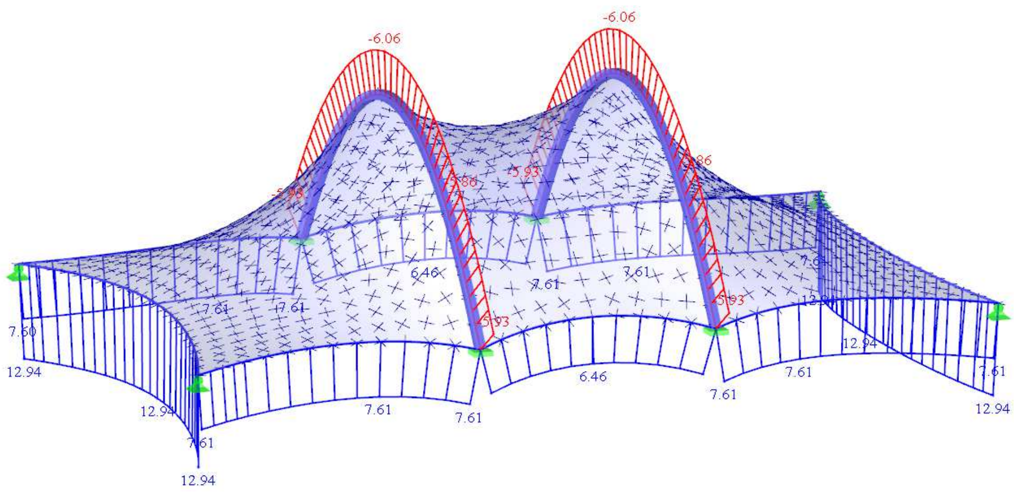

Figure 5: Equilibrium shape without beam optimization (principal forces n_1 and n_2 in membranes, normal forces N in beams and cables)

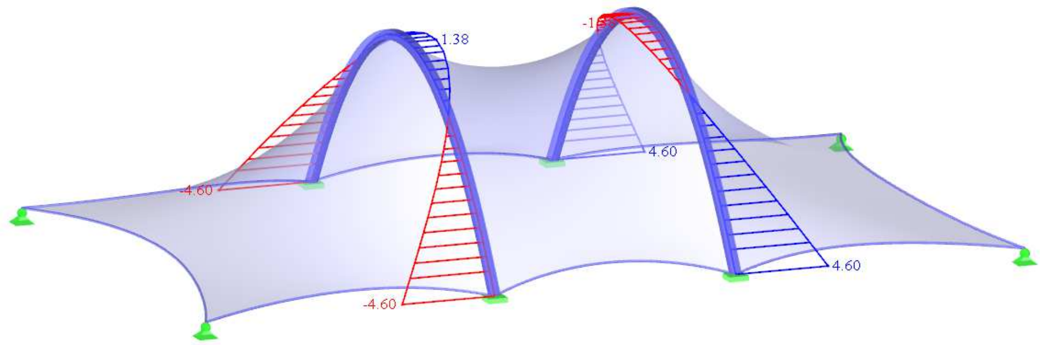

Figure 6: Equilibrium shape without beam optimization (bending moments M_z in beams)

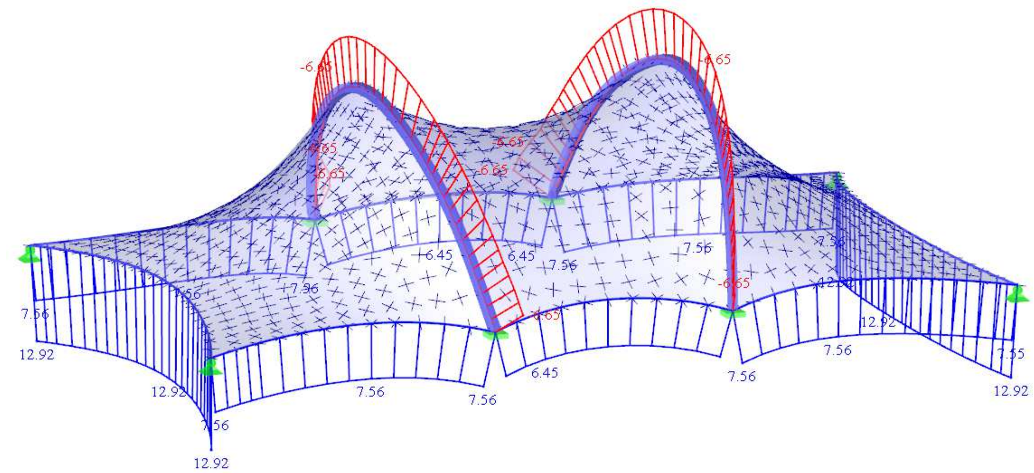

Figure 7: Equilibrium shape with beam optimization (principal forces n_1 and n_2 in membranes, normal forces N in beams and cables)

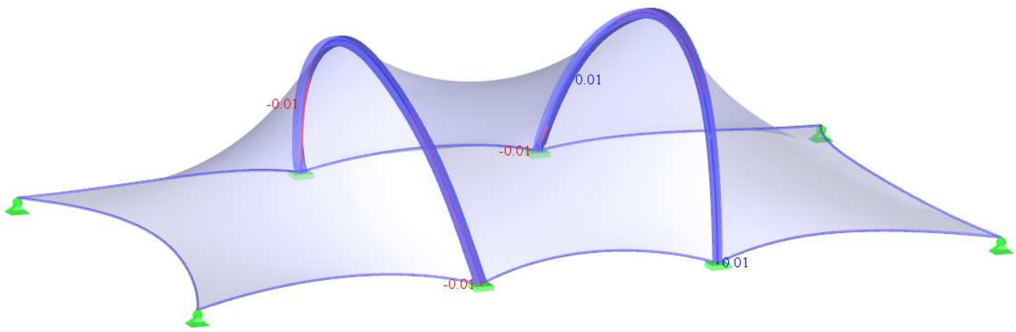

Figure 8: Equilibrium shape without beam optimization (bending moments M_z in beams)

As you can see (Figures 6 and 8), the bending moment in the second case is close to zero since the position of the arch was modified in the form-finding process. If a higher precision and a finer mesh are used, the bending moment is nearing to the limit of zero.

The form-finding process of the combined structures shows one characteristic phenomena, that there can exist more than one right solution of the same input values (Figure 9). When the initial model is made, the shape converges to the closer equilibrium position in the form-finding analysis. That's the difference between the combined structures and the exclusively tensioned or compressed structures, since they converge to the same equilibrium position from the arbitrary initial spatial position of the model.

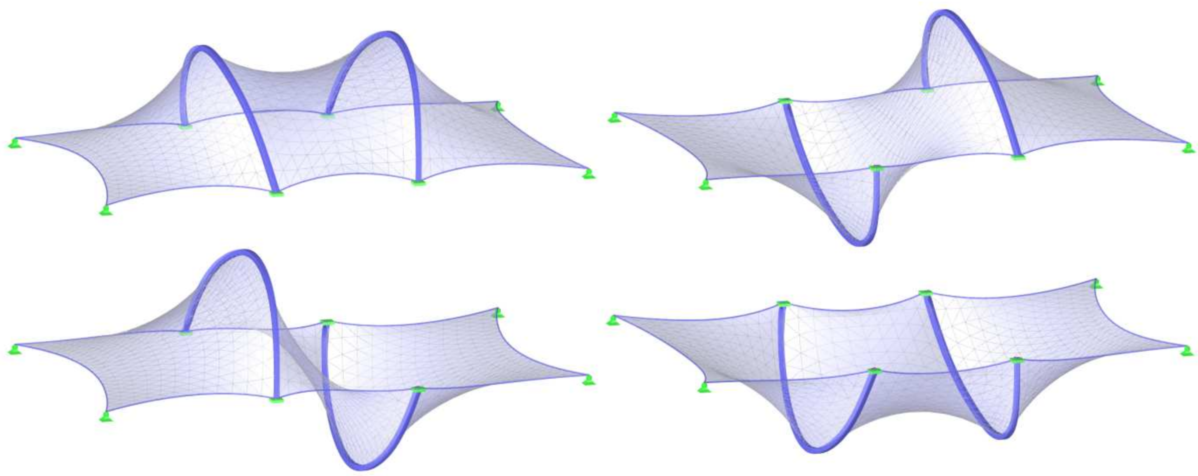

Figure 9: Existence of multiple equilibrium positions for combined structures

The last example is a concrete shell in compression supported by tensioned steel belts and with a skylight made of an ETFE cushion in the middle of this shell. Those structure components are subjected to the form-finding process and supported by concrete columns (Figures 10-11).

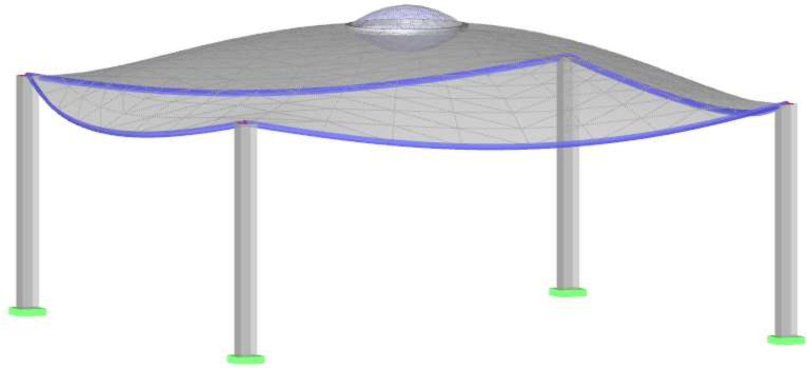

Figure 10: Equilibrium shape of a shell supported by tensioned belts on the borders with the skylight made of an ETFE cushion

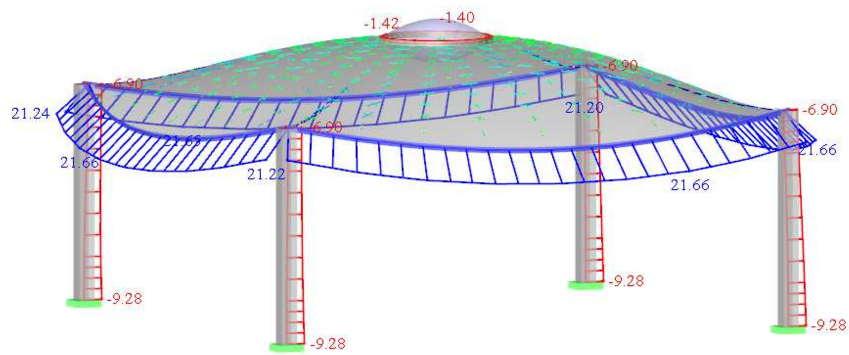

Figure 11: Compressed shell with tensioned belts and inflated cushion (principal forces n_1 and n_2 in shell and ETFE cushion, normal forces N in beams and cables)

12 CONCLUSIONS

- The paper focused on the form-finding process and the specific physical aspects and phenomena. It describes the calculation of unstable equilibrium positions of the elements in compression as well as the possibility of multiple equilibrium shapes of structures, which combine structural elements with tension and compression in the form-finding process.
- The presented examples were calculated in the RFEM software [I], where the form-finding option has been recently implemented for compressed and combined structures in cooperation of Dlubal Software and FEM consulting companies.

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